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A Deterministic Dynamic Programming Approach with Periodic Review Model for Production Planning

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Abstract- This paper presents an efficient formula for solving optimization problem in DPR model with the aid of D.D.P (Deterministic Dynamic Programming) approach. The proposed formula employs back ward recursion in which computations proceeds from last stage to first stage in a multistage decision problem. A generalized recursive equation which gives exact solution of an optimization problem is derived in this paper. The proposed method is demonstrated with a practical problem. There are two portions included in this paper. First portion gives the definitions notations and nature of DPR (Detailed Project Report) model, then follows the characteristics and idea of dynamic programming. Second portion states the production schedule type problem and analyze the nature of an optimal policy. After that, to get an efficient formula consider the d.d. programming approach, gives explanations and expression to produce the formula and shows application of the formula to the given problem by making step by step calculations till final solutions are reached. Finally summarized the resulting production schedules and point out the effectiveness of the formula.

Keywords: optimization, multistage decision, deterministic dynamic programming, optimal policy, backward and recursive relationship,

1 Introduction

Inventories means stocks of goods being held for future use or sale. Maintaining inventories is necessary for any factory dealing with physical products, including manufacturers, wholesalers and retailers. Manufacturers need inventories of the finished products awaiting shipment. The most associated with holding inventory are very large. Therefore, the cost being incurred for the storage of inventory runs into the hundreds of billions of dollars annually in many countries. Reducing storage cost by avoiding unnecessarily large inventories can enhance any firm's competitiveness. Many factories of the world have been revamping the way in which they manage their inventories. The application of optimal production schedule to minimize the total cost techniques in this scientific inventory management is providing a powerful tool for gaining a competitive edge. There are several basic consideration involved in determining an inventory policy that must be reflected in the mathematical inventory model. In the case of known demand, a deterministic inventory model would be used. Deterministic period review model is used for situations where the planning is being done for a series of periods rather than continuously. Using deterministic dynamic programming approach on this type of model is suitable to get best planning. (Chu, 1992)

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2 Objective

The objective of this paper is to explore the solution for inventory planning to minimize the total cost over the n period.

3 Literature Review

3.1 Scientific Inventory Management

This management comprises the following steps.
Formulate a mathematical model describing the behavior of the inventory system.
Seek an optimal inventory policy with respect to this model.
Searching suitable method to obtain a useful formula
Using the results of formula, apply the optimal inventory policy to signal when and how much to produce physical products. (Silver, 1995)

3.2 Dynamic Programming

Dynamic programming is a useful mathematical technique for making a sequence of interrelated decisions. It provides a systematic procedure for determining the optimal combination of decisions. In contrast to linear programming, there does not exist a standard mathematical formulation. Rather, dynamic programming is a general type of approach to problem solving, and the particular equations used must be developed to fit each situation. (Denardo, 1982)

3.3 Deterministic Dynamic Programming

Deterministic Dynamic Programming can be described as where the state at the next stage is completely determined by the state and policy decision at the current stage. Thus. At stage (n) process at some state S_n . Making policy decision, then moves to the process to some state S_{n+1}at stage (n+1). The contribution thereafter to the objective function under an optimal policy has been previously calculated. The policy decision also makes some contribution to the objective

function. Combining these two quantities in an appropriate way provides the contribution of stages n onward to the objective function.
After policy decision and function values are found for each possible of S_n, the solution procedure is ready to move back on stage. Note that for many problems (especially when the stages correspond to time period), the solution procedure must move backward and apply recursive relationship. (Bertsekas, 1987)

3.4 A Deterministic Period Review Model

In deterministic period review model, planning is to be done for the next n periods regarding how much to produce or order to replenish inventory at the beginning of each of the periods. The order to replenish inventory can involve either purchasing the units or producing them, but the latter case is far more common with applications of this model, so it will use the terminology of producing the units. The demand for the respective periods are known (but not the same in every period) and are denoted by.
d_i = Demand in period i, for i =1,2,…,n.
These demand must be met on time.
The cost included in this model are
S = setup cost for producing any units to replenish inventory at the beginning of period
c = unit cost for producing each unit
h = the holding cost for each unit left in factory at the end of period

The holding cost h is assessed only on inventory left at the end of a period. These holding costs for units that are in inventory for a portion of the period before being withdrawn to satisfy demand. However, these are fixed costs that are independent of the inventory policy and so are not relevant to the analysis. Only the variable costs that are affected by which inventory policy is chosen. To minimize the total cost over the n period is the objective. Problem should follow to explain how to minimize the total variable cost over n periods.

4 Problem statement

The sail-boat production company produces small luxury sail-boat. The company received an order from tourism Service Company for 12 sail-boats for the use of costal trip services. The order calls for two sail-boats to be delivered during first three months of a year (period 1), three more to be delivered during second three months of a year (period 2), three more during third period (period 3) and the final four during the last period (period 4). The factory setting-up the production facilities to meet the tourism company's specifications for these sail-boats requires a set-up cost of 2 million. The production factory has the capacity to produce all 12 sail-boats within three months. The holding costs is $ 0.3 million per sail-boat per period, until their scheduled delivery time. The production manager of the factory need to know to determine how many sail-boats to produce during the beginning of each of the four periods in order to minimize the total variable cost.

5 Analysis of the problem

period	demand	Set-up cost	Holding cost per unit per period.
1	2	2	0.3
2	3	2	0.3
3	3	2	0.3
4	4	2	0.3

In this case the manufacturer can produce 12 sail-boats within the first period. However, this would necessitate holding 10-sail-boats in inventory at a cost of $ 0.2 million per sail-boat per period, until their scheduled delivery times. To reduce or eliminate these substantial holding costs it may be worthwhile to produce a smaller number of these sail-boats now and then to repeat the set-up in some or all of subsequent periods to produce additional small numbers. Because of high set-up cost, there is no need to produce sail-boat every period and preferably just once. However the holding costs makes it undesirable to curry a large inventory by producing the entire demand for all periods (12-sail-boats) at the beginning.
Perhaps the best way would be an intermediate strategy where sail-boats are produced more than one but less than four-times. In general, more than once but less than (n) times. To satisfy the strategy, there are many feasible production schedules. For this model, production must make in period 1, but a decision on whether to produce for each of the other (n-1) periods. Therefore it is needed to enumerate for each of the 2^{n-1}combinations of production decisions. At that point, the question is how the optimal production schedule can be found among feasible schedules. So, a more efficient method is desirable.

6 Formulation

To get an optimal production schedule for the model is the following insight into the nature of an optimal policy.
-production rate per period must be greater than number of demands of each of period
-produce physical products, only when the inventory level is zero.
The characterization of optimal policies can be used to identify policies that are not optimal. Moreover, it implies that the only choice for the amount produced at the beginning of the i^{th}period are $0, d_i, d_i + d_{i+1}, \ldots or d_i + d_{i+1} + \cdots + d_n$, it can be exploited to obtain an efficient formula that is related to the ideology of deterministic dynamic programming approach.
Define the notation of the model,
Z_i= total variable cost of an optimal policy for period i, i+1,…,n when period (i)starts with zero inventory (before producing).
By using the deterministic dynamic programming approach

of solving backward period by period. These $Z_i$value can be found by first finding Z_n then findingZ_{n-1} ad so on. Thus, after $Z_n, Z_{n-1}, \dots, Z_{i+1}$are found then$Z_i$. (Final solution) can be found the recursive relationship.
So, the formula can be developed as
$Z^{(j)}{}_i$=$\left[Z_{j+1} + S + h\left\{\sum_{k=i}^{j}(j-k)d_{i+(j-k)}\right\}\right]$ when i=n,. . ., 1 and j = i,… ,n.
n= number of periods.
Z_i = Minimum [$Z^{(j)}{}_i$]

Note that $Z_{n+1} = 0$
The formula for solving the model consists basically of solving for $Z_n, Z_{n-1}, \dots, Z_1$ in turn. For i=1 the minimizing value of (j) Then indicates that the production in period 1 should cover the demand through period, j, so the second production will be in period j+1. For i=j+1, the new minimization value of j identifies the time interval covered by the second production, and so forth to the end. This approach will illustrate with the problem described in earlier. (Macmillan India Limited, 2006)

6.1 Application of the Formula to the Problem

$Z^{(j)}{}_i$=$\left[Z_{j+1} + S + h\left\{\sum_{k=i}^{j}(j-k)d_{i+(j-k)}\right\}\right]$
When i=n,. . . ,1 and
j = i,…,n,
n= number of periods.

$Z_{n+1} = Z_5 = 0$

i=4, j=4
$Z^{(4)}{}_4$ =$\left[Z_5 + 2 + 0.3\left\{\sum_{k=4}^{4}(j-k)d_{i+(j-k)}\right\}\right]$
=Z_5+2+0.3 $\left((4-4)d_{4+(4-4)}\right) = 2$
$Z_4 = 2$

i=3, j= 3, 4
i=3, j=3
$Z^{(3)}{}_3$=$\left[Z_4 + 2 + 0.3\left\{\sum_{k=3}^{3}(j-k)d_{i+(j-k)}\right\}\right]$
=2+2+0.3 $((0)d_3)$ =4

i=3, j=4
$Z^{(4)}{}_3$=$\left[Z_5 + 2 + 0.3\left\{\sum_{k=3}^{4}(j-k)d_{i+(j-k)}\right\}\right]$
=Z_5+2+0.3$\left((4-3)d_{3+(4-3)}\right) + (4-4)d_{3+(4-4)})$ =0+2+0.3 (d_4) =0+2+0.3(4)=3.2
Z_3 = min $(Z^{(3)}{}_3, Z^{(4)}{}_3)$ = min (4,3.2) = 3.2

i=2, j= 2, 3, 4
i=2, j=2
$Z^{(2)}{}_2$ =$\left[Z_3 + 2 + 0.3\left\{\sum_{k=2}^{2}(j-k)d_{i+(j-k)}\right\}\right]$
= 3.2 +2+0.3 $((0)d_2)$ =5.2

i=2, j= 3
$Z^{(3)}{}_2$ =$\left[Z_4 + 2 + 0.3\left\{\sum_{k=2}^{3}(j-k)d_{i+(j-k)}\right\}\right]$
$= 2 + 2 + 0.3\left((3-2)d_{2+(3-2)}\right) + (3-3)d_{2+(3-3)})$
$= 2 + 2 + 0.3(d_3 + 0d_2)$ =2+2+0.3(3) =4.9

i=2, j= 4
$Z^{(4)}{}_2$ =$\left[Z_5 + 2 + 0.3\left\{\sum_{k=2}^{4}(j-k)d_{i+(j-k)}\right\}\right]$
= 0 +2+0.3 $\left((4-2)d_{2+(4-2)} + (4-3)d_{2+(4-3)} + (4-4)d_{2+(4-4)} +\right)$
$0 + 2 + 0.3(2d_4 + d_3 + 0d_2)$
=0+2+0.3(8+3) = 5.3
Z_2 = min $(Z^{(2)}{}_2, Z^{(3)}{}_2, Z^{(4)}{}_2)$
= min (5.2,4.9,5.3) = 4.9

i=1, j= 1, 2, 3, 4
i=1, j=1
$Z^{(1)}{}_1$=$\left[Z_2 + 2 + 0.3\left\{\sum_{k=1}^{1}(j-k)d_{i+(j-k)}\right\}\right]$
=4.9+2+0.3 $((0)d_1) = 6.9$

i=1, j=2
$Z^{(2)}{}_1$=$\left[Z_3 + 2 + 0.3\left\{\sum_{k=1}^{2}(j-k)d_{i+(j-k)}\right\}\right]$
=3.2+2+0.3$((2-1)d_{1+(2-1)} + (2-2)d_{1+(2-2)})$
= 3.2 + 2 + 0.3(d_2 + $0d_1$)=3.2+2+0.3(3+0.2)=3.2+2+0.3(3)=6.1

i=1, j=3
$Z^{(3)}{}_1$ =$\left[Z_4 + 2 + 0.3\left\{\sum_{k=1}^{3}(j-k)d_{i+(j-k)}\right\}\right]$
=3.2+2+0.3$((3-1)d_{1+(3-1)} + (3-2)d_{1+(3-2)} + (3-3)d_{1+(3-3)})$
= 2 + 2 + 0.3$(2d_3 + d_2 + 0d_1)$=2+2+0.3(2×3+3+0)
=2+2+0.3(9)=6.7

i=1, j=4
$Z^{(4)}{}_1$=$\left[Z_5 + 2 + 0.3\left\{\sum_{k=1}^{4}(j-k)d_{i+(j-k)}\right\}\right]$
=0+2+0.3$((4-1)d_{1+(4-1)} + (4-2)d_{1+(4-2)} + (4-3)d_{1+(4-3)} + (4-4)d_{1+(4-4)})$
=0+2+0.3$(3d_4 + 2d_3 + d_2 + 0d_1)$
=0+2+0.3(3× 4 + 2 × 3 + 3 + 0)
=2+0.3(12+6+3)=8.3
Z_1 = min $(Z^{(1)}{}_1, Z^{(2)}{}_1, Z^{(3)}{}_1, Z^4{}_1)$ = min (6.9,6.1,6.7,8.3) = 6.1
$Z_1 = 6.1$
Based on the above calculation, Optimal Production Schedule is obtained that the production plan is to produce 5 sail-boats in period 1 and 7 sail-boats in period 3 with the total variable cost of $ 6.1 million.

7 CONCLUSION

This paper describes an optimal production schedule and inventory control policy in sail-boat Production Company. A D.D.P model with significant formula for its computation is developed and tested. The production schedule problem is complicated in three aspects; a high cost production facility, different demands for the respective periods and a substantial holding cost of storage.
The developed formula performs quite well. Estimated by practical data, the formula produces an optimal solution

(other feasible solutions can accept). Advantage of developed formula is that it is easy to explain and implement. Moreover, the application of this formula is much quicker than the full dynamic programming approach. As in dynamic programming, $Z_n, Z_{n-1}, \ldots, Z_n$ must be found before$Z_1$is obtained. However, the number of calculation is much smaller, and the number of possible production quantities is greatly reduced.

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